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NATIONAL LEVEL SCIENCE TALENT SEARCH EXAMINATION

CLASS - 11 (PCM)
Question Paper Code : UN484

KEY

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11. A	12. B	13. B	14. B	15. A	16. B	17. C	18. B	19. C	20. D
21. A	22. D	23. B	24. C	25. B	26. D	27. C	28. C	29. D	30. C
31. D	32. C	33. C	34. C	35. D	36. C	37. A	38. B	39. C	40. A
41. B	42. A	43. C	44. B	45. B	46. A	47. B	48. C	49. C	50. B
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SOLUTIONS

MATHEMATICS

01. (D) Let n(A) = number of students opted Mathematics, n(B) = number of students opted Physics and n(C) = number of students opted Chemistry

$\therefore$ n(A) = n({2, 4, 6, ... 140}) = 70

n(B) = n({3, 6, 9, ...138}) = 46

n(C) = n({5, 10, 15, ...140}) = 28

$n(A \cap B)$ = n({6, 12, 18, ...,138}) = 23

$n(B \cap C)$ = n({15, 30, 45, ...,135}) = 9

$n(A \cap C)$ = n({10, 20, 30, ...,140}) = 14

$n(A \cap B \cap C)$ = n({30, 60, 90, ...,120}) = 4

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

= 70 + 46 + 28 – 23 – 9 – 14 + 4 = 102

The number of students who did not opt for any of the three courses

$= n(A' \cap B' \cap C') = n[(A \cup B \cup C)'] = 140 - n(A \cup B \cup C) = 140 - 102 = 38$

02. (C) $\cos[\pi^2]x + \cos[-\pi^2]x$

$= \cos 9x + \cos(-10)x$

$= \cos 9x + \cos 10x$

f(0) = cos 0 + cos0 = 2,

$$= f\left(\frac{\pi}{4}\right) = \cos\frac{9\pi}{4} + \cos\frac{5\pi}{2} = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

$$= f\left(\frac{\pi}{2}\right) = \cos\frac{9\pi}{2} + \cos 5\pi = 0 - 1 = -1,$$

$f(\pi) = \cos 9\pi + \cos 10\pi = -1 + 1 = 0$

03. (B) $z^{\frac{-1}{3}} = a + ib$

$\Rightarrow \overline{z} = (a + ib)^3$

$\Rightarrow x - iy = a^3 + 3a^2bi + 3ab^2i^2 + i^3b^3$

$\Rightarrow x - iy = (a^3 - 3ab^2) - i(b^3 - 3a^2b)$

$\Rightarrow x = a^3 - 3ab^2, y = b^3 - 3a^2b$

$$\Rightarrow \frac{x}{a} = a^2 - 3b^2, \frac{y}{b} = b^2 - 3a^2$$

$$\Rightarrow \frac{x}{a} + \frac{y}{b} = -2a^2 - 2b^2 = -2\left(a^2 + b^2\right)$$

$$\Rightarrow \frac{1}{a^2 + b^2}\left(\frac{x}{a} + \frac{y}{b}\right) = -2$$

04. (B) (i) Suppose $x - 3 \geq 0 \Rightarrow x \geq 3$

Given equation is

$x^2 + x - 3 = 4 \Rightarrow x^2 + x - 7 = 0$

$$\Rightarrow x = \frac{-1 \pm \sqrt{1+28}}{2} = \frac{-1 \pm \sqrt{29}}{2},$$ which are not possible

(ii) Suppose $x - 3 < 0 \Rightarrow x < 3$

Given equation is

$x^2 - (x - 3) = 4 \Rightarrow x^2 - x - 1 = 0$

$$\Rightarrow x = \frac{1 \pm \sqrt{1+4}}{2} = \frac{-1 \pm \sqrt{5}}{2} < 3$$

Sum of roots of the equation

$$= \frac{1+\sqrt{5}}{2} + \frac{1-\sqrt{5}}{2} = \frac{1}{2} + \frac{1}{2} = 1$$

05. (C) $$\frac{{}^{n+2}C_6}{{}^{n-2}P_2} = 11$$

$$\Rightarrow \frac{(n+2)!}{6!(n-4)!} \times \frac{(n-4)!}{(n-2)!} = 11$$

$$\Rightarrow \frac{(n+2)(n+1)(n)(n-1)}{720} = 11$$

$\Rightarrow (n + 2)(n + 1)n(n - 1) = 11 \times 10 \times 9 \times 8$

$\Rightarrow n = 9 \Rightarrow n^2 + 3n - 108 = 0$

06. (C) The number of questions of the sections in the selection may be of the following types.

	1st section (5)	2nd section (3)	3rd section (2)
Type 1:	4	1	1
2 :	3	2	1
3 :	2	3	1
4 :	3	1	2
5 :	2	2	2
6 :	1	3	2

$$= {}^5C_4 \times {}^3C_1 \times {}^2C_1 + {}^5C_3 \times {}^3C_2 \times {}^2C_1 + {}^5C_2 \times {}^3C_3 \times {}^2C_1$$
$$+ {}^5C_3 \times {}^3C_1 \times {}^2C_2 + {}^5C_2 \times {}^3C_2 \times {}^2C_2 + {}^5C_1 \times {}^3C_3 \times {}^2C_2$$

$= 5 \times 3 \times 2 + 10 \times 3 \times 2 + 10 \times 1 \times 2$

$+ 10 \times 3 \times 1 + 10 \times 3 \times 1 + 5 \times 1 \times 1$

$=30 + 60 + 20 + 30 + 30 + 5 = 175$

07. (A) $A = 1^2 + 2.2^2 + 3^2 + ... + 2.20^2$

$= (1^2 + 2^2 + ... + 20^2) + (2^2 + 4^2 + ... + 20^2)$

$$= \frac{20 \times 21 \times 41}{6} + \frac{4 \times 10 \times 11 \times 21}{6}$$

$= 2870 + 1540 = 4410$

$B = 1^2 + 2.2^2 + 3^2 + ... + 2.40^2$

$= (1^2 + 2^2 + ... + 40^2) + (2^2 + 4^2 + ...+40^2)$

$$= \frac{40 + 41 \times 81}{6} + \frac{4 \times 20 \times 21 \times 41}{6}$$

$= 22140 + 11480 = 33620$

$100\lambda = B - 2A = 33620 - 8820 = 24800$

$= 100 \times 248$

$\Rightarrow \lambda = 248$

08. (C) $S = (1 + x)^{1000} + 2x(1 + x)^{999} + ... +1001x^{1000}$

$$\frac{x}{1+x}S = x(1+x)^{999} + 2x^2(1+x)^{998} + ... + \frac{1001x^{1001}}{1+x}$$

$$\left(1 - \frac{x}{1+x}\right)S = (1+x)^{1000} + x(1+x)^{999} + x^2(1+x)^{998} + ... + x^{1000} - \frac{1001x^{1001}}{1+x}$$

$$\Rightarrow \frac{1}{1+x}S = (1+x)^{1000} + x(1+x)^{999} + + ... + x^{1000} - \frac{1001x^{1001}}{1+x}$$

S = $(1+x)^{1001} + x(1+x)^{1000} + \ldots + x^{1000}(1+x) - 1001x^{1001}$

$$= \frac{(1+x)^{1001}\left[1-\left(\frac{x}{1+x}\right)^{1001}\right]}{1-\frac{x}{1+x}} - 1001x^{1001}$$

$= (1+x)^{1002} - x^{1001}(1+x) - 1001x^{1001}$

Coefficient $x^{50} = {}^{1002}C_{50}$

09. (B) $\tan^2 x - \tan^4 x + \tan^8 x + \tan^4 x - \ldots$

$$= \frac{\tan^2 x}{1-(-\tan^2 x)} = \frac{\tan^2 x}{\sec^2 x} = \sin^2 x$$

$\therefore$ $y = \exp\{(\tan^2 x - \tan^4 x + \tan^6 x - \tan^8 x + \ldots)\log_e 16\}$

$= \exp\{(\sin^2 x)\log_e 16\}$

$= \exp\{(\log_e(16^{\sin^2 x})\} = 16^{\sin^2 x}$

y satisfies $x^3 - 3x + 2 = 0$

$\Rightarrow y = 1$ or $y = 2$

$\Rightarrow 16^{\sin^2 x} = 1 \text{ (or) } 16^{\sin^2 x} = 2$

since $0 < x < \frac{\pi}{4}$, $0 < \sin x < \frac{1}{\sqrt{2}}$

$\Rightarrow 0 < \sin^2 x < \frac{1}{2}$

$\therefore 16^{\sin^2 x} = 1$ is not possible

Thus, $16^{\sin x} = 2$

$\sin^2 x = \frac{1}{4}$, Thus, $\cos^2 x + \cos^4 x$

$= (1 - \sin^2 x) + (1 - \sin^2 x)^2 = \frac{21}{16}$

10. (A) $\lim\limits_{x\to 3} \frac{\sqrt{3x}-3}{\sqrt{2x-4}-\sqrt{2}}$

$$\lim_{x\to 3}\left[\frac{\sqrt{3x}-3}{\sqrt{2x-4}-\sqrt{2}} \times \frac{\sqrt{3x}+3}{\sqrt{3x}+3} \times \frac{\sqrt{2x-4}+\sqrt{2}}{\sqrt{2x-4}+\sqrt{2}}\right]$$

$$= \lim_{x\to 3} \frac{(3x-9)\left(\sqrt{2x-4}+\sqrt{2}\right)}{(2x-4-2)\left(\sqrt{3x}+3\right)}$$

$$= \lim_{x\to 3}\left[\frac{3}{2}\left(\frac{\sqrt{2x-4}+\sqrt{2}}{\sqrt{3x}+3}\right)\right] = \frac{3}{2} \times \frac{2\sqrt{2}}{6} = \frac{1}{\sqrt{2}}$$

11. (A) $y^{\cos x} = x^{\sin y}$

$\Rightarrow \cos x \log y = \sin y \log x$

$$\Rightarrow \frac{\cos x}{y}\frac{dy}{dx} + \log y(-\sin x)$$

$$= \frac{\sin y}{x} + \log x \cos y \frac{dy}{dx}$$

$$\Rightarrow \left(\frac{\cos x}{y} - \log x \cos y\right)\frac{dy}{dx}$$

$$= \frac{\sin y}{x} + \log y \sin x\, s$$

$$\Rightarrow \frac{dy}{dx} = \frac{y(\sin y + x \log y \sin x)}{x(\cos x - y \log x \cos y)}$$

12. (B) The numbers are

1, 1 + d, 1 + 2d,...., 1 + 100d

The numbers are in A.P

Then mean

= 51st term = 1 + 50d = $\overline{x}$ (say)

Mean deviation(M.D) = $\frac{1}{n}\sum_{i=1}^{101}\left|x_i - \overline{x}\right|$

$$= \frac{1}{101}[50d + 49d + 48d + \ldots + d + 0 + d + 2d + \ldots + 50d]$$

$$= \frac{1}{101} \times 2d(1 + 2 + \ldots + 50)$$

$$= \frac{1}{101} \times 2d\frac{50 \times 51}{2} = \frac{50 \times 51}{101}d$$

But M.D = 255

Given $\Rightarrow \frac{50 \times 51}{101}d = 255$

$$\Rightarrow d = \frac{101 \times 255}{50 \times 51}$$

$$= \frac{101 \times 255}{2550} = 10.1$$

13. (B) Let n be the number of children in each family. 3 tickets can be distributed in family B in ${}^{n}C_3$ ways.

3 tickets can be distributed in both the families in ${}^{2n}C_3$ ways.

Given $\frac{{}^{n}C_3}{{}^{2n}C_3} = \frac{1}{12}$

$\Rightarrow 12\,{}^{n}C_3 = {}^{2n}C_3$

$\Rightarrow 12n(n - 1)(n - 2)$

$= 2n(2n - 1)(2n - 2)$

$\Rightarrow 3(n - 2) = 2n - 1$

$n = 5$

14. (B) A(1) A(4) + A(2) A(5)

$= (\sin\alpha + \cos\alpha)(\sin^4\alpha + \cos^4\alpha) + (\sin^2\alpha + \cos^2\alpha)(\sin^5\alpha + \cos^5\alpha)$

$= (\sin\alpha + \cos\alpha)(\sin^4\alpha + \cos^4\alpha) + \sin^5\alpha + \cos^5\alpha$

A(1) A(6) + A(2) A(3)

$= (\sin\alpha + \cos\alpha)(\sin^6\alpha + \cos^6\alpha) + (\sin^2\alpha + \cos^2\alpha)(\sin^3\alpha + \cos^3\alpha)$

$= (\sin\alpha + \cos\alpha)(1 - \cos^2\alpha)\sin^4\alpha + (\sin\alpha + \cos\alpha)(1 - \sin^2\alpha)\cos^4\alpha + \sin^3\alpha + \cos^3\alpha$

$= (\sin\alpha + \cos\alpha)(\sin^4\alpha + \cos^4\alpha) - (\sin\alpha + \cos\alpha)\cos^2\alpha\sin^4\alpha - (\sin\alpha + \cos\alpha)\sin^2\alpha\cos^4\alpha + \sin^3\alpha + \cos^3\alpha$

$= (\sin\alpha + \cos\alpha)(\sin^4\alpha + \cos^4\alpha) - (\sin\alpha + \cos\alpha)\sin^2\alpha\cos^2\alpha + \sin^3\alpha + \cos^3\alpha$

$= (\sin\alpha + \cos\alpha)(\sin^4\alpha + \cos^4\alpha) - \sin^3\alpha\cos^2\alpha - \sin^2\alpha\cos^3\alpha + \sin^3\alpha + \cos^3\alpha$

$= (\sin\alpha + \cos\alpha)(\sin^4\alpha + \cos^4\alpha) + \sin^3\alpha(1 - \cos^2\alpha) + \cos^3\alpha(1 - \sin^2\alpha)$

$= (\sin\alpha + \cos\alpha)(\sin^4\alpha + \cos^4\alpha) + \sin^5\alpha + \cos^5\alpha$

= A(1) A(4) + A(2) A(5)

15. (A) $\cot(\alpha + \beta) = 0 \Rightarrow \cos(\alpha + \beta) = 0$

$\Rightarrow \cos\alpha\cos\beta - \sin\alpha\sin\beta = 0$

$\Rightarrow \cos\alpha\cos\beta = \sin\alpha\sin\beta$

Now $\sin(\alpha + 2\beta) = \sin(\alpha + \beta + \beta)$

$= \sin(\alpha + \beta)\cos\beta + \cos(\alpha + \beta)\sin\beta$

$= \sin(\alpha + \beta)\cos\beta$ $[\because \cos(\alpha + \beta) = 0]$

$= (\sin\alpha\cos\beta + \cos\alpha\sin\beta)\cos\beta = \sin\alpha\cos^2\beta + \sin\beta\cos\alpha\cos\beta$

$= \sin\alpha\cos^2\beta + \sin\beta\sin\alpha\sin\beta$

$[\because \cos\alpha\cos\beta = \sin\alpha\sin\beta]$

$= \sin\alpha[\cos^2\beta + \sin^2\beta]$

$= \sin\alpha(1) = \sin\alpha$

16. (B) Put $\tan\frac{x}{2} = t$

$3\sin x + 4\cos x = 5$

$$\Rightarrow 3\left(\frac{2t}{1+t^2}\right) + 4\left(\frac{1-t^2}{1+t^2}\right) = 5$$

$\Rightarrow 6t + 4(1 - t^2) = 5(1 + t^2)$

$\Rightarrow 9t^2 - 6t + 1 = 0$

$\Rightarrow 6t - 9t^2 = 1$

$$6\tan\frac{x}{2} - 9\tan^2\frac{x}{2} = 1$$

17. (C) $0 < A < B < \frac{\pi}{4} \Rightarrow A + B, A - B \in Q_1$

$\cos(A + B) = \frac{11}{61} \Rightarrow \sin(A + B) = \frac{60}{61}$;

$\sin(A - B) =$

$\frac{24}{25} \Rightarrow \cos(A - B) = \frac{7}{25}$

$\sin 2A + \sin 2B$

$$= 2\sin(A+B)\cos(A-B) = 2 \times \frac{60}{61} \times \frac{7}{25} = \frac{168}{305}$$

18. (B) $\tan\left(\frac{A}{2}\right)\tan\left(\frac{C}{2}\right)$

$$=\sqrt{\frac{(s-b)(s-c)}{s(s-a)}}\sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$$

$$\Rightarrow \frac{5}{6}\times\frac{2}{5}=\frac{s-b}{s}$$

$\Rightarrow$ s = 3s – 3b

2s = 3b $\Rightarrow$ a + b + c = 3b

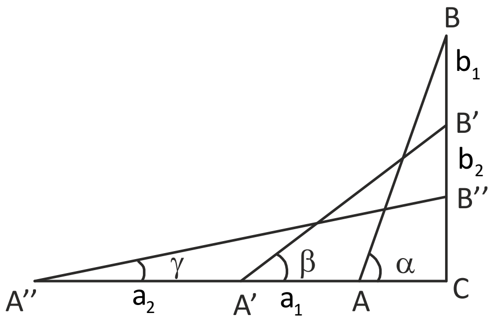
$\Rightarrow$ a + c = 2b

$\Rightarrow$ a, b, c are in A.P

19. (C) If l be the length of the ladder, then

$a_1 = l\cos\beta - l\cos\alpha$ and b_1

$= l\sin\alpha - l\sin\beta$



Also, $a_2 = l\cos\gamma - \cos\beta$ and b_2

$= l\sin\beta - l\sin\gamma$

$$\therefore \frac{a_1}{b_1}=\frac{2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)}{2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)}$$

$$\Rightarrow \frac{a_1}{b_1}=\tan\left(\frac{\alpha+\beta}{2}\right)$$

Similarly, $\frac{a_2}{b_2}=\tan\left(\frac{\beta+\gamma}{2}\right)$

Since, $a_1a_2 = b_1b_2$ (given)

$$\Rightarrow \frac{a_1}{b_1}=\frac{b_2}{a_2}\Rightarrow \tan\left(\frac{\alpha+\beta}{2}\right)=\frac{1}{\tan\left(\frac{\beta+\gamma}{2}\right)}$$

$$\Rightarrow \tan\left(\frac{\alpha+\beta}{2}\right)=\cot\left(\frac{\beta+\gamma}{2}\right)$$

$$=\tan\left[\frac{\pi}{2}-\left(\frac{\beta-\gamma}{2}\right)\right]$$

$$\therefore \frac{\alpha+\beta}{2}=\frac{\pi}{2}-\left(\frac{\beta+r}{2}\right)$$

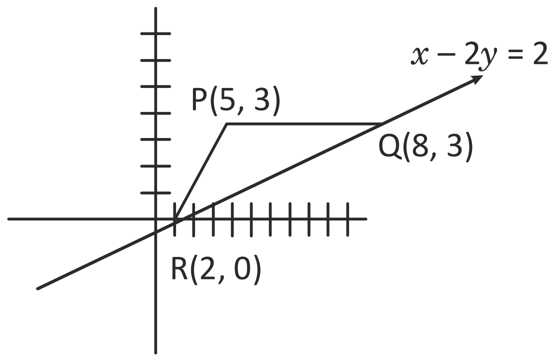
$$\therefore \frac{\alpha+\beta}{2}+\frac{\beta+r}{2}=\frac{\pi}{2}$$

$$\frac{\alpha+\beta+\beta+\gamma}{2}=\frac{\pi}{2}$$

$\Rightarrow \alpha + \beta + \gamma = \pi - \beta < \pi$

20. (D) Equation of RQ is $x - 2y = 2 \rightarrow$ (i)

at $y = 0$, $x = 2$ [R (2, 0)]



as PQ is parallel to x, y-coordinates of Q is also 3

Putting value of y in equation (i), we get Q(8, 3)

Centroid of ΔPQR

$$=\left(\frac{8+5+2}{3},\frac{3+3}{3}\right)=(5,2)$$

Only ($2x - 5y = 0$) satisfy the given co-ordinates

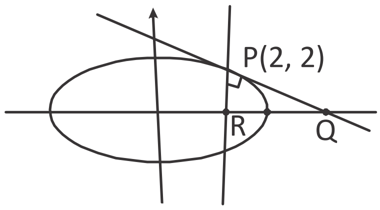
21. (A)

p	q	$p\to q$	$p\wedge(p\to q)$	$(p\wedge(p\to q))\to q$	$q\to p\wedge(p\to q)$
T	T	T	T	T	T
T	F	F	F	T	T
F	T	T	F	T	F
F	F	T	F	T	T

22. (D) $3x^2 + 5y^2 = 32$

$$\Rightarrow \frac{3x^2}{32} + \frac{5y^2}{32} = 1$$

Tangent on the ellipse at P is



$$\frac{3(2)x}{32} + \frac{5(2)y}{32} = 1 \Rightarrow \frac{3x}{16} + \frac{5y}{16} = 1$$

$\therefore$ co-ordinates of Q will be $\left(\frac{16}{3}, 0\right)$

Now, normal at P is

$$\frac{32}{3(2)} - \frac{32y}{5(2)} = \frac{32}{3} - \frac{32}{5}$$

$\therefore$ co-ordinates of R will be $\left(\frac{4}{5}, 0\right)$

Hence, area of $\Delta PQR = \frac{1}{2}(PQ)(PR)$

$$= \frac{1}{2}\sqrt{\frac{136}{9}}\sqrt{\frac{136}{25}} = \frac{68}{15}$$

23. (B) $f(x) = \left(\frac{3}{5}\right)^x + \left(\frac{4}{5}\right)^x - 1$

Put $f(x) = 0$

$$\Rightarrow 0 = \left(\frac{3}{5}\right)^x + \left(\frac{4}{5}\right)^x - 1$$

$$\Rightarrow \left(\frac{3}{5}\right)^x + \left(\frac{4}{5}\right)^x = 1$$

$\Rightarrow 3^x + 4^x = 5^x$

For $x = 1$

$3^1 + 4^1 > 5^1$

For $x = 3$

$3^3 + 4^3 = 91 < 5^3$

Only for $x = 2$, equation (i) Satisfy

So, only one solution ($x = 2$)

24. (C) Let l_1, m_1, n_1 and l_2, m_2, n_2 be the d.c of line 1 and 2 respectively, then as given

$l_1 + m_1 + n_1 = 0$

and $l_2 + m_2 + n_2 = 0$

and $l_1^2 + m_1^2 - n_1^2 = 0$ and

$l_2^2 + m_2^2 - n_2^2 = 0$

($\because l + m + n = 0$ and $l^2 + m^2 - n^2 = 0$)

Angle between lines, θ is

$\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2 \qquad \rightarrow (1)$

As given $l^2 + m^2 = n^2$ and $l + m = -n$

$\Rightarrow (-n)^2 - 2lm = n^2$

$\Rightarrow 2lm = 0$ or $lm = 0$

So $l_1 m_1 = 0$, $l_2 m_2 = 0$

If $l_1 = 0$, $m_1 \neq 0$ then $l_1 m_2 = 0$

If $m_1 = 0$, $l_1 \neq 0$ then $l_2 m_1 = 0$

If $l_2 = 0$, $m_2 \neq 0$ then $l_2 m_1 = 0$

If $m_2 = 0$, $l_2 \neq 0$ then $l_1 m_2 = 0$

Also $l_1 l_2 = 0$ and $m_1 m_2 = 0$

$l^2 + m^2 - n^2 = l^2 + m^2 + n^2 - 2n^2 = 0$

$$\Rightarrow 1 - 2n^2 = 0 \Rightarrow n = \pm\frac{1}{\sqrt{2}}$$

$$\therefore n_1 = \pm\frac{1}{\sqrt{2}},\ n_2 = \pm\frac{1}{\sqrt{2}}$$

$$\therefore \cos\theta = \frac{1}{2} \Rightarrow \theta = 60° \text{(acute angle)}$$

25. (B) Given x, y, z are in GP

$$\therefore \frac{y}{x} = \frac{3}{y} \qquad \text{....... (1)}$$

Given $a^x = b^y = c^z = k$

$a^x = k \Rightarrow \log_a k = x$

similarly $\log_b k = y$

$\log_c k = z$

$$\therefore \frac{\log_b k}{\log_a k} = \frac{\log_c k}{\log_b k} \quad [\because \text{from eq } (1)]$$

$$\Rightarrow \log_b^a = \log_c^b$$

PHYSICS

26. (D) In instantaneous speed and instantaneous velocity $\Delta t \rightarrow 0$, therefore,

|displacement| = distance

A particle may have variable velocity by changing direction even when magnitude is not changing.

27. (C) m = 400 g = 0.4 kg

h_1 = 5 m, F = 100 N, h_2 = 20 m

t = ?, g = 10 m/s^2

Now, $v_1 = \sqrt{2gh_1} = \sqrt{2\times10\times5} = 10$ m/s

$$v_2 = \sqrt{2gh_2} = \sqrt{2\times10\times20} = 20 \text{ m/s}$$

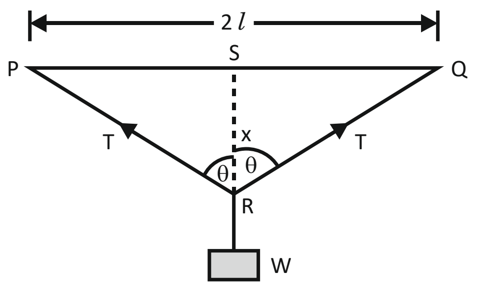
As, F × t = m[v_2 – (– v_1)]

∴ 100 × t = 0.4 (20 + 10) = 12.0

$$t = \frac{12.0}{100} = 0.12 \text{ s}$$

28. (C) In the given figure, the increase in length

Δl = (PR + RQ) – PQ = 2 PR – PQ



$$= 2(l^2 + x^2)^{\frac{1}{2}} - 2l = 2l\left(1 + \frac{x^2}{l^2}\right) - 2l$$

$$= 2l\left[1 + \frac{1}{2}\frac{x^2}{l^2}\right] - 2l$$

$$= \frac{x^2}{l} \text{(By Binomial Theorem)}$$

$$\therefore \text{ Strain} = \frac{\Delta l}{2l} = \frac{x^2}{2l^2}$$

29. (D) As no torque is being applied, angular momentum remains constant

$$I_1\omega_1 = I_2\omega_2$$

$$(mr_1^2)\omega_1 = (mr_2^2)\omega_2$$

$$\frac{\omega_1}{\omega_2} = \frac{r_2^2}{r_1^2}$$

30. (C) Here, c = $M^0 L^1 T^{-1}$ = × 10^8 m/s

g = $M^0 L^1 T^{-2}$ = 10 m/s^2

p = $M^1 L^{-1} T^{-2}$ = 10^5 N/m^2

$$\frac{c}{g} = \frac{LT^{-1}}{LT^{-2}} = T = \frac{3\times10^8}{10} = 3\times10^7 \text{ s}$$

$$\text{From } c = \frac{L}{T} = 3\times10^8$$

L = 3 × 10^8 T

= 3 × 10^8 × 3 × 10^7

= 9 × 10^{15} m

From $M^1 L^{-1} T^{-2}$ = 10^5

M = 10^5 × $L^1 T^2$

= 10^5 × 9 × 10^{15} (3 × 10^7)^2

= 81 × 10^{34} kg

31. (D) As the acceleration of the projectile is always downward (because of its gravitational acceleration), the vertical speed decreases as the projectile rises and increases as the projectile falls.

Option (A) is false because the acceleration vector is straight down, but the velocity is never straight up or straight down. It would be true of the vertical component of the velocity, but not of the total velocity.

Option (B) is false because the projectile still has its horizontal velocity at the top of the trajectory.

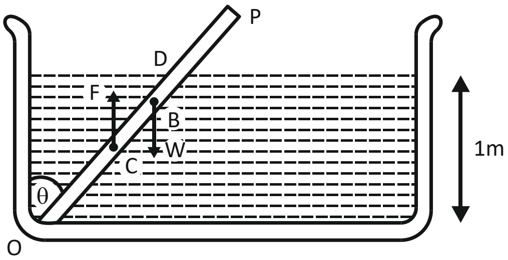
Option (C) is false because the vertical component of the velocity changes, even if the horizontal component does not, so the total speed changes.

32. (C)

$$\Delta U = U_f - U_i = \frac{-GMm}{R+\frac{R}{5}} - \left[\frac{-GMm}{R}\right] = \frac{GMm}{R}\left[1-\frac{5}{6}\right]$$

$$\text{or } \Delta U = \frac{GMm}{6R} = \frac{mR}{6}\left(\frac{GM}{R^2}\right) = \frac{1}{6}mgR = \frac{5}{6}mgh$$

33. (C) Let B be the centre of gravity of rod and C be the middle point of the length of rod in water, OD = 1.0 secθ



Then $OC = \frac{OD}{2} = \frac{1}{2}\sec\theta$. If A is the area of cross-section of rod, then mass of rod

$$= 2.0 = 2 \times A \times 500 \text{ or } A = \frac{1}{500}m^2$$

Upthrust on rod F

$$= (1.0\sec\theta)\left(\frac{1}{500}\right) \times 1000 \times 10 = 20\sec\theta$$

Weight of rod, W = 2 × 10 = 20 N.

For rotational equilbrium of rod, net torque about O should be zero.

∴ F × (OC sin θ) = W × (OB sin θ)

or $20\sec\theta \times \left(\frac{1}{2}\sec\theta\right)\sin\theta = 20 \times (1.0\sin\theta)$

or $\sec^2\theta = 2$ or $\sec\theta = \sqrt{2} = \sec 45°$

or $\theta = 45°$

∴ F = 20 sec 45° = 20 $\sqrt{2}$ N

For vertical equilibrium of the rod, force exerted by the hinge on the rod will be

= (20$\sqrt{2}$ – 20) N downwards

= 8.28 N or 8.3 N downwards

34. (C) The work done by external force F is equal to the increase in potential energy of the bob.

Therefore, $W_F = \Delta U = mgL(1 - \cos\theta)$

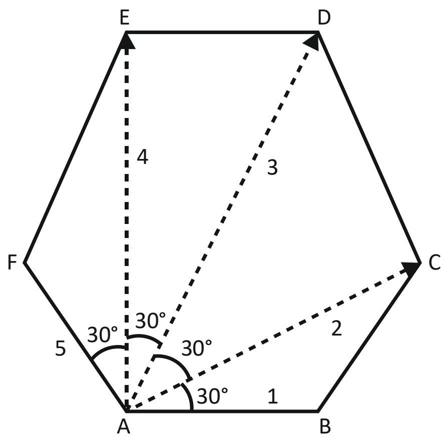
35. (D) Growth of ice in a pond is conduction process governed by the relation,

$$t = \frac{\rho L}{K\theta}\frac{y^2}{2}$$

The ratio of times for thickness of ice from 0 to y; y to 2 y = 1 : 3

∴ Time taken to increase the thickness from 1 cm to 2 cm is equal to 3 × 7 = 21 hours.

36. (C) The lines AE and AB are perpendicular to each other and are taken as the Y-axis and X-axis respectively. The vectors are resolved along the X and the Y axis. The algebraic sum of the x component is



x = 1 + 2 cos 30° + 3 cos 60° – 5 cos 60°

$$= 1 + 2 \times \frac{\sqrt{3}}{2} + 3 \times \frac{1}{2} - \frac{5}{2} = \sqrt{3} \text{ unit}$$

The algebraic sum of the y component is

= 2 sin 30 + 3 sin 60 + 4 + 5 sin 60

$$= 2 \times \frac{1}{2} + 3 \times \frac{\sqrt{3}}{2} + 4 + 5 \times \frac{\sqrt{3}}{2}$$

= (5 + 4$\sqrt{3}$) unit

The resultant

$R = \sqrt{x^2 + y^2} = \sqrt{3 + (5 + 4\sqrt{3})^2}$ = 12.05 unit

Let θ be the angle between the resultant and the side AB

$\tan\theta = \frac{y}{x} = \frac{5 + 4\sqrt{3}}{\sqrt{3}}$ = 6.887, θ = 81° 45'

37. (A) Process 1 is isobaric (P = Constant) expansion. Hence, temperature of gas will increase.

∴ ΔU_1 = Positive

Process 2 is an isothermal process

∴ $\Delta U_2 = 0$

Process 3 is an adiabatic expansion.

Hence, temperature of gas will fall.

∴ ΔU_3 = Negative

∴ $\Delta U_1 > \Delta U_2 > \Delta U_3$

38. (B) As, $T = 2\pi\sqrt{\frac{(R+x)^3}{MG}}$

or $\frac{T^2}{4\pi^2} = \frac{(R+x)^3}{MG}$

Centripetal acceleration,

$$a = \frac{GM}{(R+x)^2}$$

or $\frac{(R+x)^2}{GM} = \frac{1}{a}$

or $(R+x) = \frac{T^2}{4\pi^2} \times a$

$$= \left(\frac{5.26 \times 10^3}{2\pi}\right)^2 \times 9.32$$

= 160 × 10^3 m = 160 km

39. (C) l = 4.234 m, b = 1.005 m, thickness h = 2.01 cm = 0.0201 m

Total area = 2[lb + lh + bh]

= 2[4.234 × 1.005 + 4.234 × 0.0201 + 1.005 × 0.0201]

= 8.7209 m^2 = 8.72 m^2

Correcting to three significant figures as there are only three significant figures in thickness,

Volume = l × b × h

= 4.234 × 1.005 × 0.0201 = 0.085528 m^3

= 0.0855 m^3

Volume is corrected upto three significant figures.

40. (A) $K.E. = \frac{1}{2}mv^2 = \frac{1}{2} \times 9 \times 10^{-31}(10^3)^2$

= 4.5 × 10^{-25} J

From $v^2 - u^2 = 2as$, $v^2 = 2as$,

$$a = \frac{v^2}{2s} = \frac{(10^3)^2}{2 \times 10^{-1}}$$

F = ma = 9 × 10^{-31} (0.5 × 10^7) N

$$= \frac{4.5 \times 10^{-24}}{9.8} \text{kg wt} = 0.46 \times 10^{-24} \text{ kg wt}$$

CHEMISTRY

41. (B) Energy absorbed in the ionization of 1 mole of Mg (g) to Mg^+ (g) = 750 kJ.

Energy left unconsumed = 1200 – 750 = 450 kJ

This energy is required to convert Mg^+ (g) to Mg^{2+} (g)

Thus, % of Mg^{2+} (g) = $\frac{450}{1450} \times \frac{100}{1} = 31\%$

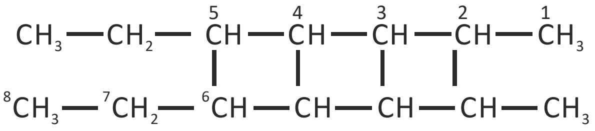
and % of Mg^+ (g) = 100 – 31 = 69%

42. (A) Reaction (b) is double of (a) and reverse of (a).

Hence, $K_2 = \frac{1}{K_1^2}$ or $K_1^2 = \frac{1}{K_2}$

43. (C) SF_4 (sp^3d, trigonal bipyramidal with one equatorial position occupied by 1 lone pair) CF_4 (sp^3, tetrahedral, no lone pair), XeF_4 (sp^3d^2, square planar, two lone pairs).

44. (B) Supplying requisite number of H-atoms, the given hydrocarbon becomes :



5-Ethyl-2,3,4-trimethyloctane

45. (B) $n(H_2O) = \dfrac{\text{Mass of water}}{\text{Molar mass of water}}$

$$= \frac{12\text{ g}}{18\text{ g mol}^{-1}} = 0.67\text{ mol}$$

$$n(C_2H_5OH) = \frac{\text{Mass of ethanol}}{\text{Molar mass of ethanol}}$$

$$= \frac{92\text{ g}}{46\text{ g mol}^{-1}} = 2.00\text{ mol}$$

$$n(CH_3COOH) = \frac{\text{Mass of acetic acid}}{\text{Molar mass of acetic acid}}$$

$$= \frac{108\text{ g}}{60\text{ g mol}^{-1}} = 1.80\text{ mol}$$

So, Total number of moles in the solutions, n_{total} = (0.67 + 2.00 + 1.80) mol = 4.47 mol

Therefore,

$$X_{water} = \frac{n(H_2O)}{n_{total}} = \frac{0.67\text{ mol}}{4.47\text{ mol}} = 0.15$$

$$X_{ethanol} = \frac{n(C_2H_5OH)}{n_{total}} = \frac{2.00\text{ mol}}{4.47\text{ mol}} = 0.45$$

$$X_{acetic\ acid} = \frac{n(CH_3COOH)}{n_{total}} = \frac{1.80\text{ mol}}{4.47\text{ mol}} = 0.40$$

Thus, the mole fraction of water in the given mixture is 0.15.

46. (A) The balanced chemical equation is

$$2MnO_4^- + 5C_2O_4^{2-} + 16H^+ \longrightarrow 2Mn^{2+} + 10CO_2 + 8H_2O$$

47. (B) As the hydrogen atom has only one orbit containing only one electron, the ionisation potential of the ground state of the hydrogen atom is the energy of the electron of the first orbit, i.e.,

$E_1 = -2.17 \times 10^{-11}$ erg.

Thus, $E_2 = \dfrac{E}{n^2}$ ..... (Eqn. 5)

$$= -\frac{2.17 \times 10^{-11}}{2^2} \quad \text{..... (n = 2)}$$

$\therefore$ Energy of the radiation emitted,

$$\Delta E = E_2 - E_1$$

$$= \frac{-2.17 \times 10^{-11}}{2^2} - (-2.17 \times 10^{-11})$$

$= 1.627 \times 10^{-11}$ erg.

We know that $\Delta E = h\nu = \dfrac{hc}{\lambda}$

Thus, $\dfrac{hc}{\lambda} = 1.627 \times 10^{-11}$

$$\lambda = \frac{6.62 \times 10^{-27} \times 3 \times 10^{10}}{1.627 \times 10^{-11}} = 1.22 \times 10^{-5}\text{ cm}$$

= 1220 Å.

48. (C) $RCOOH + NaHCO_3 \rightarrow RCOONa + H_2O + CO_2$

or

$$RCOOH + HCO_3^- \rightleftharpoons RCOO^- + H_2O + CO_2$$

As equilibrium goes in the forward direction, the conjugate base, $RCOO^-$ is more stable than RCOOH.

49. (C) $\%\text{ of N in } C_3H_9N_3 = \dfrac{42}{87} \times 100 = 48.27$

$$\%\text{ of N in } C_2H_8N_2 = \frac{28}{60} \times 100 = 46.66$$

$$\%\text{ of N in } C_6H_{12}N_4 = \frac{56}{140} \times 100 = 40.00$$

Thus, the decreasing percentage of N is :

$C_3H_9N_3 > C_2H_8N_2 > C_6H_{12}N_4$

50. (B) The entropy of vaporisation ($\Delta_{vap}S$) of a liquid is given by,

$$\Delta_{vap}S = \frac{\Delta_{vap}H}{T_{vap}} = \frac{42.4\,\text{kJ mol}^{-1}}{(78.4+273)\,\text{K}} = \frac{42.4}{351.4}\text{kJ K}^{-1}\text{ mol}^{-1}$$

$$\Delta_{vap}S = \frac{42.4}{351.4}\times 1000\,\text{J K}^{-1}\text{mol}^{-1} = 120.7\text{ J K}^{-1}\text{ mol}^{-1}$$

51. (B) $2\,Al + 3H_2SO_4 \rightarrow Al_2\,(SO_4)_3 + 3\,H_2$

2×27 g = 54 g $\qquad$ $3 \times 2 = 6$ g

2 g H_2 = 18 g Al

$Zn + H_2SO_4 \rightarrow ZnSO_4 + H_2$

65 g $\qquad$ 2 g

2 g H_2 = 65 g Zn

$Fe + H_2SO_4 \rightarrow FeSO_4 + H_2$

56 g $\qquad$ 2 g

2 g H_2 = 56 g Fe

52. (D) μ (100% ionic) = q × d

$= 4.8 \times 10^{-10}$ cm × 1.3×10^{-8} cm = 6.24 D

$\therefore$ % ionic character = $\frac{\mu_{obs}}{\mu_{100\%\ ionic}} \times 100$

$$= \frac{1.03}{6.24} \times 100 = 16.5\,\%\text{ or }17\%$$

53. (D) When intensity x is doubled, number of electrons emitted per second y is also doubled but average energy z of photoelectrons emitted remains the same.

54. (D) KOH is a strong alkali and is completely dissociated into the constituent ions,

$KOH + H_2O\text{ (excess)} \rightarrow K^+(aq) + OH^-(aq)$

In a solution having pH = 12, the hydrogen ion concentration is given by the equation,

pH = – log [H+]

12 = – log [H+]

or $[H^+] = 10^{-12}$ mol L^{-1}

As the ionic product of water should have a fixed value, hence at 25° C.

$K_w = 1.0 \times 10^{-14}$

So, $1.0 \times 10^{-14} = [H^+]\,[OH^-]$

This gives, $[OH^-] = \frac{1.0\times10^{-14}}{10^{-12}}$

$= 1.0 \times 10^{-2}$ mol L^{-2}

As KOH is completely dissociated, hence

$[KOH] = [OH^-] = 1.0 \times 10^{-2}$ mol L^{-2}

Molar mass of KOH = (39 + 16 + 1) g mol^{-1} = 56 g mol^{-1}

Then, Conc. of KOH = 1.0×10^{-2} mol L^{-1} × 56 g mol^{-1} = 0.56 g L^{-1}

Thus, 0.56 g of KOH should be dissolved per litre of the solution to obtain a solution of pH 12.

55. (B) Statements (i), (iii) and (iv) are correct

In the isoelectronic series, all isoelectronic anions belong to the same period and cations to the next period.

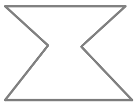
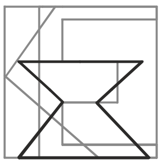
CRITICAL THINKING

56. (C) Option A is incorrect: The passage nowhere refers to or makes an implied reference to Ramsar Convention.

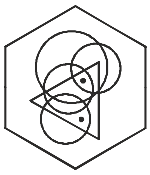
Option B is incorrect: The passage is suggesting for the opposite as to what is mentioned in option B. Instead of focusing on modernizing and augmenting the water system(i.e. augmenting the water supply), policies must focus on the source of such water i.e. it must try to strengthen the capacity of ecological systems. However, as per the given passage, public policies are doing just the opposite.

Option C is correct: The first statement clearly states that "One of the biggest ironies, around water is that it comes from rivers and other wetlands. Yet it is seen as divorced from them. While water is used as a resource, public policy does not always grasp that is a part of the natural ecosystem."

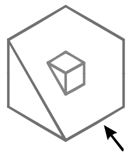
Option D is incorrect: While the statement given in option D is correct in its own merit, it is out of context with respect to the given passage, as the author does not state or indicate towards any such measure. Also, the statement is rather narrow in approach, as compared to the overall broader tone of the author.

57. (D)  = 

58. (D) So that one dot appears in the triangle and one circle; and the other dot appears in the triangle and three circles;



59. (C) The central and state governments share the cost of the Midday Meal Scheme, with the center providing 60 percent and the states 40 percent. The central government provides grains and financing for other food. Costs for facilities, transportation, and labor is shared by the federal and state governments.

60. (B) 

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